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# SPECOL:

## A colorimetric measurement system

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# SPECOL: A COLORIMETRIC MEASUREMENT SYSTEM

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## Summary

*This Report describes a newly constructed colour measurement system comprising a light detector and a microcomputer. The light detector is a monochromator which can be tuned over the entire wavelength range of visible light so that spectral energy distributions of all types of light sources can be measured. The computer has direct control over the monochromator and can derive the chromaticity coordinates of the light source by calculation.*

*The equipment has many modes of operation and can measure the properties of absorptive and reflective filters as well as those of self luminous sources such as lamps and television displays. For measurements of tricolour television displays the performance of the display can be analysed for conformity with the relevant EBU specification. Using this equipment, measurements can be made rapidly and with good accuracy.*

*For the newcomer to colour science the Report includes an introduction to the subject, together with an explanation of the mathematical processes used.*


Issued under the Authority of

M.E.A. Mobbs

Head of Research Department

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SPECOL: A COLORIMETRIC MEASUREMENT SYSTEM

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# SPECOL: A COLORIMETRIC MEASUREMENT SYSTEM

A. Roberts, B.Eng.

## 1. INTRODUCTION

This Report describes a simple but accurate apparatus for the measurement of colour. In order to assist the inexperienced reader, however, there will first be an introduction to the concepts of colour and some ways of measuring it. This is not intended to be a comprehensive treatise on the subject since there are many books available; some suitable examples are listed in the bibliography.

### 1.1. What Is Colour?

The sensation experienced by an observer of a coloured scene is conditioned by the optical system of the eye and by neurological processing performed in the retina and brain. These processes are not yet fully understood but a simple model describes the eye as an optical system which projects light onto its back layer (the retina). A mosaic of light-sensitive cells converts the light to electrical nerve impulses. There are two kinds of cells, called rods and cones after their physical shapes, which are sensitive to brightness and colour respectively. The rod cells give monochrome vision at low light levels, such as moonlight and below, and are responsible for the impression of sharpness. The cones are responsible for colour vision and are of three types, each having a light absorbing pigment which makes it sensitive to reddish, greenish or bluish light. The outputs of these cells are coded to provide a brightness and two colour difference pathways in the brain, in much the same way as a television signal is coded. The population density of the cells is not uniform; at the centre of the retina there are no rod cells at all.

Thus the sensation of colour depends on an analysis of light from the scene, performed by the three types of colour sensitive cells. The characteristics of the pigments are well known, having been measured by several observers over many years. A specific scene may not give the same impression to two observers if the pigments in their cone cells are not identical, since the absorption of the cells will not be identical; such problems are best described as defects of colour vision, and as such are beyond the scope of this Report. Other factors which affect colour vision are chromatic aberration and fluorescence; these are also outside the scope of this Report but for a further treatment of the subject the reader may consult some of the references in the bibliography.

### 1.2. Why Do We Need to Measure Colour?

The need to measure colour arises principally from applications in which colour is to be reproduced, such as in film, printing, fabrics, television, paints, dyes etc. Particularly in television the need arises because the camera is a colour analyser, the display is a colour synthesizer, and the two should match each other. It is this matching which is of great importance, since the television system must reproduce the image of the scene as accurately as possible. In practice some distortions may be deliberately introduced in order, for example, to compensate for the reduction of perceived contrast under television viewing conditions. Nevertheless, these distortions should be precisely controlled, which implies that they should only be applied once the rest of the system has been designed for the most accurate reproduction. The colorimeter described in this Report was developed primarily to measure the performance of the display, and future work is expected to continue in order to develop a similar system for camera colorimetry measurement.

### 1.3. How Do We Quantify Colour?

The commonly accepted method for the quantification of colour is to assign to it three subjective attributes; viz. brightness, hue and colourfulness.

***Brightness*** describes the sensation according to which a part of the scene appears to exhibit more or less light. Allied to this is lightness which is the brightness related to an equally lit area of white.

***Hue*** describes the sensation according to which a part of the scene appears to be similar to one or a combination of two of the 'rainbow' colours, i.e. red, orange, yellow, green, blue and purple. This leads to ***chromatic colours*** and ***achromatic colours*** in which colours have or have not a hue, respectively.

***Colourfulness*** describes the sensation according to which a part of the scene appears to exhibit more or less of a chromatic colour. Allied to this is ***saturation*** which is the colourfulness related to its brightness, and ***perceived chroma*** which is the colourfulness related to the brightness of a similarly lit area of white.

Since these attributes are subjective, they may have different values for two observers of the same

colour. Such a system is of little value for scientific measurements and so objective correlates have been derived, which can be measured scientifically. These are luminance, dominant wavelength and purity.

***Luminance*** is the correlate of brightness, and is the luminous intensity per unit area in a given direction (measured in candelas per square metre, $cd/m^2$). Allied to it is ***luminance factor*** which is the correlate of lightness, being the ratio of the luminance of a body to that of an identically lit perfect diffuser.

***Dominant wavelength*** is the correlate of hue and is the wavelength of the monochromatic light stimulus which, when suitably mixed with an achromatic light stimulus (white), precisely matches the colour in question.

***Colorimetric purity*** is the correlate of saturation and is the ratio of the luminances of the dominant wavelength stimulus and the achromatic stimulus required to produce a colour match as described above.

While describing terms, it may be of interest to note that ***colour temperature*** is the temperature of a Planckian (black body) radiator having the same chromaticity as a given colour, and ***correlated colour temperature*** is the temperature of such a radiator whose perceived colour most closely matches that of a given colour. The term ***chromaticity*** is also relevant; this is described in Section 2.1.

#### 1.4. How Do We Measure Colour?

The basic requirement of a colour measurement system is to derive, numerically, the amounts of each of three or more primary light sources needed to produce an optical match to the colour in question. An instrument designed to fulfil this function is called a ***colorimeter***.

The simplest colorimeter for a reflective test sample consists of a white illuminator for the sample and three coloured lamps illuminating a reflective white reference. The colour can then be described numerically in terms of the brightnesses of the lamps when they have been adjusted to produce a colour match. These are called the ***tristimulus*** values. Obviously the values depend on the choice of colours used for the illuminating lamps. A standard was set up by the Commission Internationale d'Eclairage (CIE) in 1931 which used monochromatic (single spectrum line) sources located at wavelengths of 700, 546.1 and 435.8 nm respectively for red, green and blue; the units were scaled such that white was represented by equal quantities of these sources.

Using such a colorimeter the results can give negative tristimulus values. This seemingly ridiculous situation arises when a particular colour cannot be matched by an additive mixture of the primaries. Under these conditions the solution is to add one or more of the primaries to the sample instead of the white reference, thus reducing its saturation and giving a negative value for the tristimulus when a match has been achieved. A better solution is to use a different set of primaries which always result in positive values, and such a set was adopted by the CIE, also in 1931, using new but nonreal primaries called $X$, $Y$ and $Z$.

The $XYZ$ system uses primaries which cannot exist; therefore the simple colorimeter described above cannot be realised, and another method is required. The solution is to synthesize the primaries mathematically. This requires first measuring the spectral reflectance distribution function of the colour in question and then multiplying it by the ***colour matching functions*** of the three primaries in order to obtain the tristimulus values. Effectively each wavelength component of the colour sample is matched by the equivalent amounts of the $X$, $Y$ and $Z$ primaries and then these individual amounts are added together to give the final match. The colour matching functions are referred to as $\bar{x}$, $\bar{y}$ and $\bar{z}$, and are defined as the tristimulus values of the three primary colours required to match monochromatic lights of equal radiant energy, expressed as functions of their wavelength. The CIE $Y$ primary was chosen for convenience to represent luminance only, and thus it is identical with the ***luminous efficiency function***, $V_\lambda$. The $X$ and $Z$ primaries have zero luminance and contain only colour information. The luminous efficiency function results from the additive law for brightness which is the basic principle of photometry and states that the condition for a brightness match between two different colours is

$$\int P'_\lambda \,.\, V_\lambda \,.\, d\lambda = \int P''_\lambda \,.\, V_\lambda \,.\, d\lambda$$

where $\lambda$ denotes wavelength, $P'_\lambda$ and $P''_\lambda$ are the spectral energy distributions of the two colours, $V_\lambda$ is a function of wavelength, and the integral extends over the whole visible range of wavelengths. Since the colorimeter is now an analytical machine, all the foregoing descriptions apply equally to transmissive, reflective or radiant sample colours.

All that is required is that the spectral power distribution function of the sample is measured and processed using the colour matching functions in order to produce the tristimulus values for that sample. Such a colorimeter is the subject of this Report.

### 1.5. SpeCol, the New Colorimeter

The equipment performs colorimetric measurements by first measuring the spectral power distribution of the test colour and then calculating, using data tables, the tristimulus values for the colour. Subsequent calculations produce chromaticity coordinates and can deduce the colour matching ability of a set of three test colours when used as the primaries for a television display. For some time, Research Department has used this technique for the measurement of monitors etc. The equipment used was based on a digital voltmeter with off-line analysis on the Departmental computer. With the increasing availability of inexpensive microcomputers with adequate processing power, however, it was decided to simplify and improve the measurement equipment. The new apparatus comprises a monochromator, microcomputer and small interface unit, and is described in more detail in Section 4.

## 2. BASIC COLORIMETRY

### 2.1. Colour Spaces

A colour space is a means of representing colours as vectors in a three dimensional volume. Two spaces which typify the systems most commonly used are the Munsell samples system and the CIE chromaticity system. There are many other possible spaces but, they can all be derived from one of these by processes of mathematical transformation.

The Munsell system[1] describes colours by their appearance, and as such is a subjective system. It uses lightness, hue and chroma as three, mutually exclusive, descriptors and represents each colour as a point within a cylinder whose vertical axis represents lightness, radius represents chroma and rotation represents hue as shown in Fig. 1. Thus black is at the bottom and white at the top, colours increase in chroma with distance from the central axis and hue changes with angular position. Each radial plane from this solid, taken from the central axis to the outer surface, is a plane of constant hue and is constructed by mixing together, in varying quantities, a chromatic paint with black and white. Thus the colour solid is not truly a cylinder but each radial plane is a curve with the outermost point representing the pure chromatic paint colour. The principal publication, the Munsell Book of Color, is a book of sample colours produced in this way. The obvious merit of this system is that each plane contains only shades of one hue, with all the reproducible range of chroma and lightness displayed together. A disadvantage is that it is not possible to produce, by painting, the full range of perceivable colours.

The ***CIE*** chromaticity system[2] is specifically formulated to be able to represent mathematically any colour by a set of three orthogonal numbers which can be thought of as existing in a solid. Thus it is similar to the Munsell system but it uses objective rather than subjective values. The vertical axis is taken to be luminance and thus the whole gamut of colours can be represented in the horizontal plane, described by ***chromaticity coordinates***. These can be derived by normalising the tristimulus values, $X$, $Y$ and $Z$, obtained from a colorimeter, to obtain three other parameters, $x$, $y$ and $z$, thus:

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

Since these values always sum to unity it is sufficient to describe any colour using only two of them. Commonly $x$ and $y$ are used, but the tristimulus value $Y$ must be retained if the luminance information is also required.

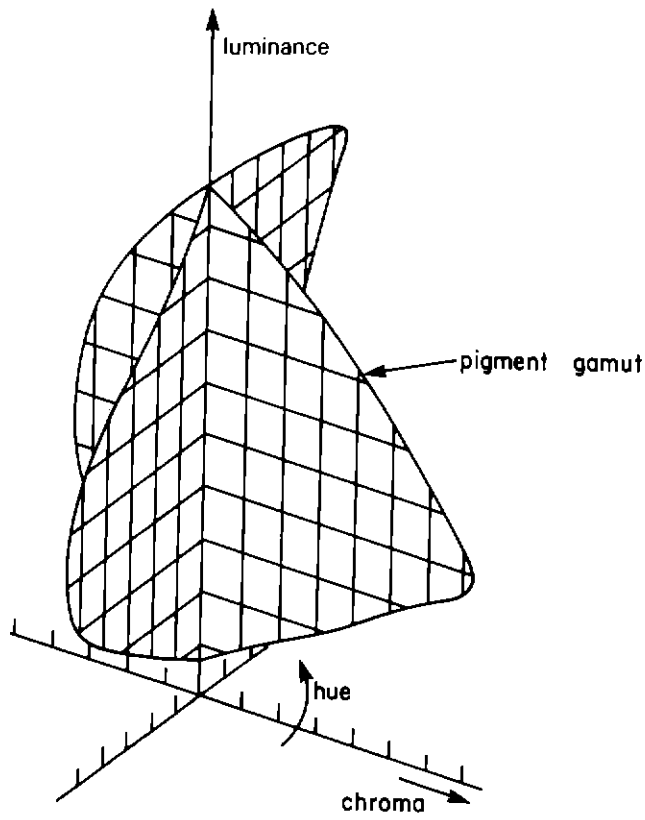


*Fig. 1 – The Munsell colour space.*

The chromaticity plane of this system is shown in Fig. 2, with the locus of chromatic colours drawn over the range of wavelengths from 400 to 700 nm (the luminance is represented by an ordinate normal to the paper). The line joining the ends of the curve represents the locus of the most saturated purples that can be generated. From this it can readily be seen that the primaries are nonreal since the values $x = 1$ and $y = 1$ both occur well outside this curve, which contains all the perceivable colours, as is the point $x = y = 0$, where $z = 1$. For comparison a chromaticity plane using coordinates derived from the CIE red, green and blue primaries is shown in Fig. 3, where the primaries lie upon the locus of chromatic colours, and it is evident that negative amounts of red light are required to describe a large range of colours. The reader is referred to Appendix A for a more detailed examination of this subject.

### 2.2. Simple Tristimulus Colorimetry

As was mentioned in Section 1.4, the simplest colorimeter floods a white patch with three coloured lights in order to produce an optical match with the test colour under white light. The tristimulus values are the amounts of each coloured light required to produce a match, scaled so that equal amounts match white. The method is not best suited to measurements of real colours, but for the sake of completeness Fig. 4 shows the general arrangement that might be used. Note that the red lamp can also be used to illuminate the test sample in order to match the large range of cyan and green colours for which a negative red value results. For reasons of ***metamerism*** (see Section 3.1 for an explanation of the term) it is essential to use monochromatic light sources. Tungsten lamps with chromatic liquid filters have been used and lasers are also suitable.

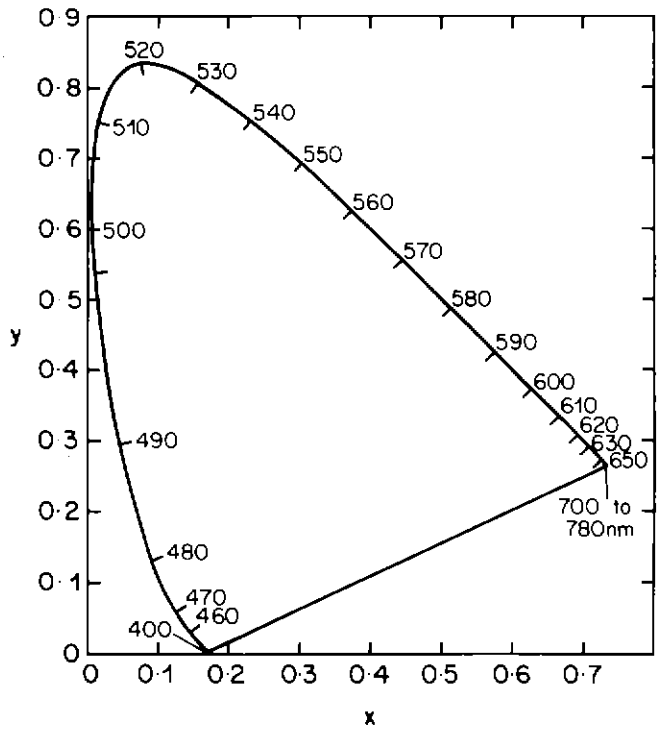


*Fig. 2 – The CIE 1931 (x—y) chromaticity diagram, showing the spectrum locus from 400 to 700 nm and the line of the most saturated purples.*

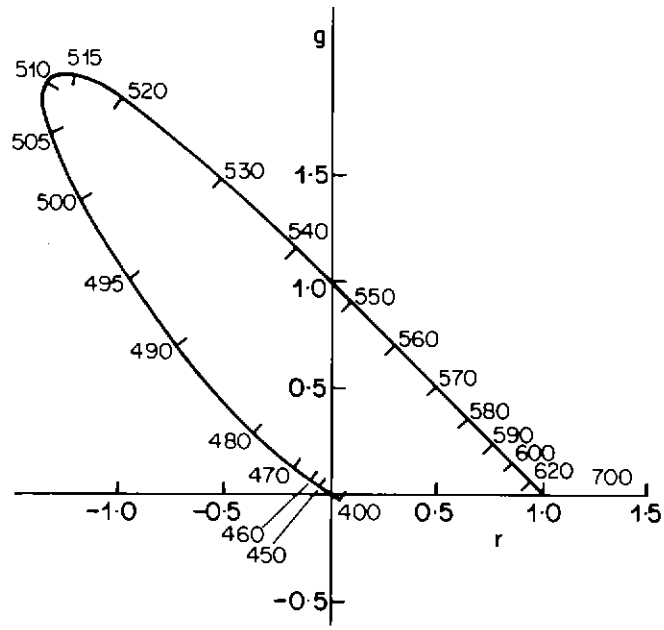


*Fig. 3 – The r—g chromaticity diagram, showing the spectrum locus from 400–700 nm.*

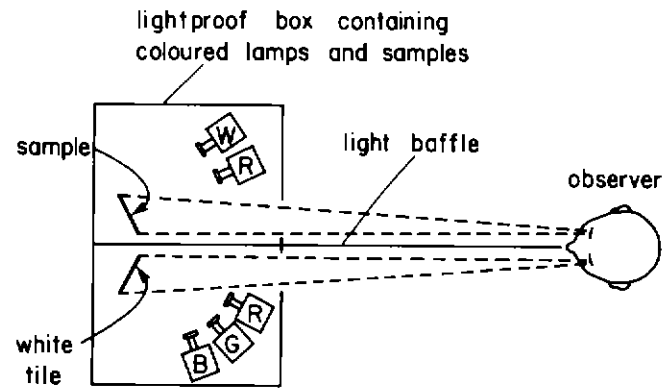


*Fig. 4 – A simple colorimeter, using red green and blue lamps for colour matching.*

### 2.3. Spectral Colorimetric Measurement

This much more general method of measuring colour involves measuring, wavelength by wavelength ($\lambda$), the light energy coming from the test colour. The measured light may result from reflectance of white light, transmission of white light by filtering, or direct radiation of light. The spectral light energy distribution thus obtained ($P_\lambda$) may be converted to tristimulus values mathematically by multiplying it by each of the colour matching functions ($\bar{x}$, $\bar{y}$ and $\bar{z}$) in turn and integrating the resultant curves. Chromaticity coordinates may then be obtained by normalisation.

The colour matching functions, representing the amount of each primary required to match the individual wavelengths in the test colour, are shown in Fig. 5. They are the results of measurement of the colour vision of a number of observers before 1931 by Wright[3] and others and were accepted by the CIE as the definition of the ***CIE Standard Observer***; they are thus uniquely defined and provide an ideal basis for an objective measurement system.

The means of measuring the spectral light energy distribution can be by ***spectroradiometry*** or ***spectrophotometry***. A spectroradiometer measures the spectral energy distribution linearly, subsequent calculations being necessary to derive the chromaticity coordinates. A spectrophotometer measures the tristimulus values or chromaticity coordinates directly using filters designed to replicate the colour matching functions. The relative advantages and disadvantages of these two techniques are considered in more detail in the following section.

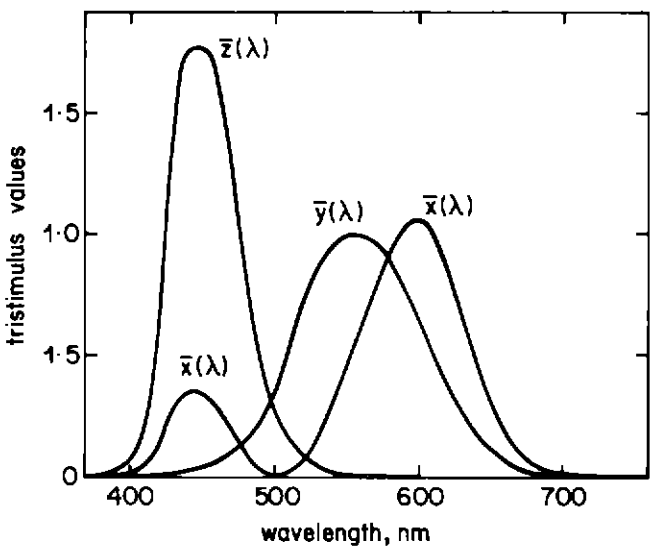


*Fig. 5 – The colour matching functions (1931 2° standard observer).*

## 3. MEASUREMENT TECHNIQUES

### 3.1. Spectrophotometry

Spectrophotometric equipment usually comprises three light sensitive cells, each covered with a colour filter chosen so that the spectral sensitivity of the three cells mimics the colour matching functions ($\bar{x}$, $\bar{y}$ and $\bar{z}$). Thus the electrical outputs from the cells give a direct reading of the tristimulus values ($X$, $Y$ and $Z$) of the test colour, from which the chromaticity coordinates may be obtained. Clearly it is essential that the colour filters precisely match the colour matching functions, and that the instrument should be calibrated by measurement of a standard white source. Such instruments give instant readings and are valuable for day-to-day non-critical measurement, but cannot be expected to give highly accurate results. It is also possible to realise such a colorimeter using four or two cells giving better or worse performance respectively.

The greatest merit of the spectrophotometer is that it can be a small, hand held device which produces instant readings.[7] Its greatest disadvantage is that its accuracy depends on the smoothness of the spectrum it is measuring. With a smooth spectrum, small errors in the filter responses are of little importance, but with light sources whose spectra do not vary smoothly, such as red phosphors or discharge lamps, the light energy may be in the form of only a few narrow-band peaks occurring at wavelengths where even small filter errors can result in meaningless tristimulus values. Thus different values may be obtained for ***metameric*** colours. Two colours are metameric if they have different spectral characteristics but the same chromaticity coordinates (the opposite case is ***isomeric*** colours, which always have the same spectral characteristics). For this reason, spectrophotometers should be used with care.

### 3.2. Spectroradiometry

Spectroradiometric equipment comprises a means of measuring the light energy, wavelength by wavelength, across the whole spectral range and then some form of data processor which calculates the tristimulus values by multiplication of this spectrum with the colour matching functions. Thus, provided that the bandwidth of the wavelength scanning device is accurately controlled, and its gain characteristic can be measured, accurate results are obtainable for both metameric and isomeric colours.

## 4. THE SPECOL EQUIPMENT

SpeCol, the spectroradiometric colorimeter described in this Report, comprises a ***monochromator*** and a microcomputer with minimal interfacing electronics. A monochromator is a wavelength scanning device, which, with a photomultiplier, can measure the spectral energy of a light source at intervals down to 1 nm using a rotating grating or prism as the frequency/wavelength selective device. The general arrangement of such a monochromator is shown in Fig. 6. Incoming light is focused onto a vertical grating via an obscuring slit and concave mirror. The grating disperses the light into a rainbow which is detected via another concave mirror and slit. The grating is rotated by the action of a stepping motor to select any particular wavelength and, in the device used, the mechanical arrangement was designed such that one step of the motor produced a wavelength change of 1 nm. The speed of scanning is generally determined by the sensitivity of the light detector. If a highly sensitive detector is used then the grating can be rotated sufficiently rapidly for a representation of the spectral energy to be shown on an oscilloscope. In the equipment described here, however, it was considered important to reduce the effects of noise in the detector as much as possible, in order to achieve the highest accuracy. Thus relatively low detector bandwidths were used, with several readings being averaged at each wavelength to minimise effects such as induced electrical hum etc.

The complete apparatus is shown in a photograph (Fig. 7) and functional block diagram (Fig. 8). In addition to the monochromator with its associated photomultiplier detector it comprises a microcomputer with a small interface unit. This computer directly controls the stepping motor to rotate the monochromator grating, and measures the photocurrent signal from the photomultiplier via an electrical filter to reduce the effects of noise. A high voltage supply to the photomultiplier is the only other system component. The equipment is thus small, self-contained and relatively transportable.

The software for the system is extensive, allowing the user many options for measurement and display of results. It was designed for ease of operation, and extensive use has been made of choices from a menu display of permissible options. Program overlaying techniques have been adopted so that the whole program need not reside in computer memory at once and the user is not necessarily aware of the operations required to move between parts of the program. Measurements are stored on floppy discs and a catalogue of them can be built for archival purposes; typically over 600 measurements can be stored on one 5.25 inch floppy disc.

The equipment can perform measurements and analysis of light sources and reflective or absorptive filters, and produce a variety of output. It can be

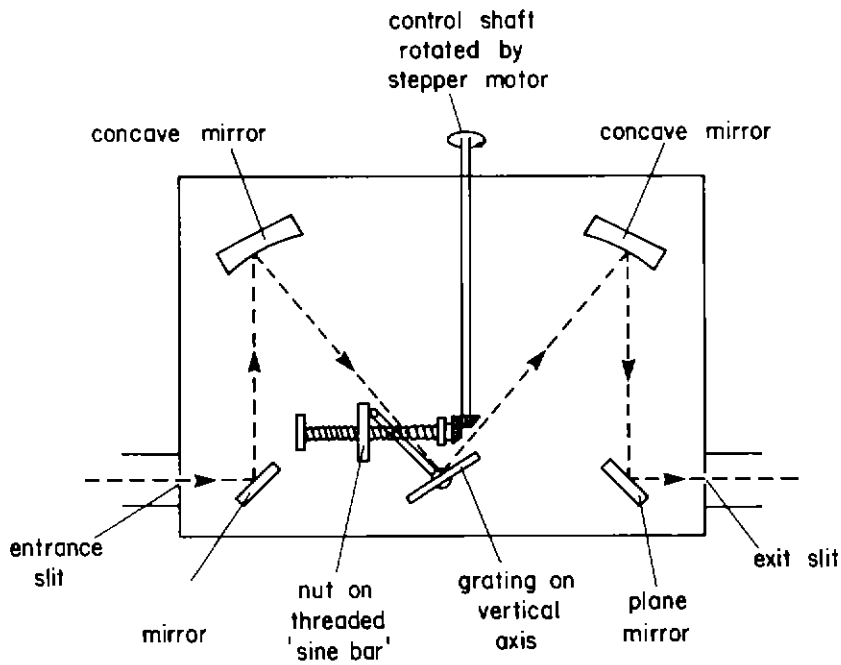


*Fig. 6 – Diagram of the optical path through a monochromator.*

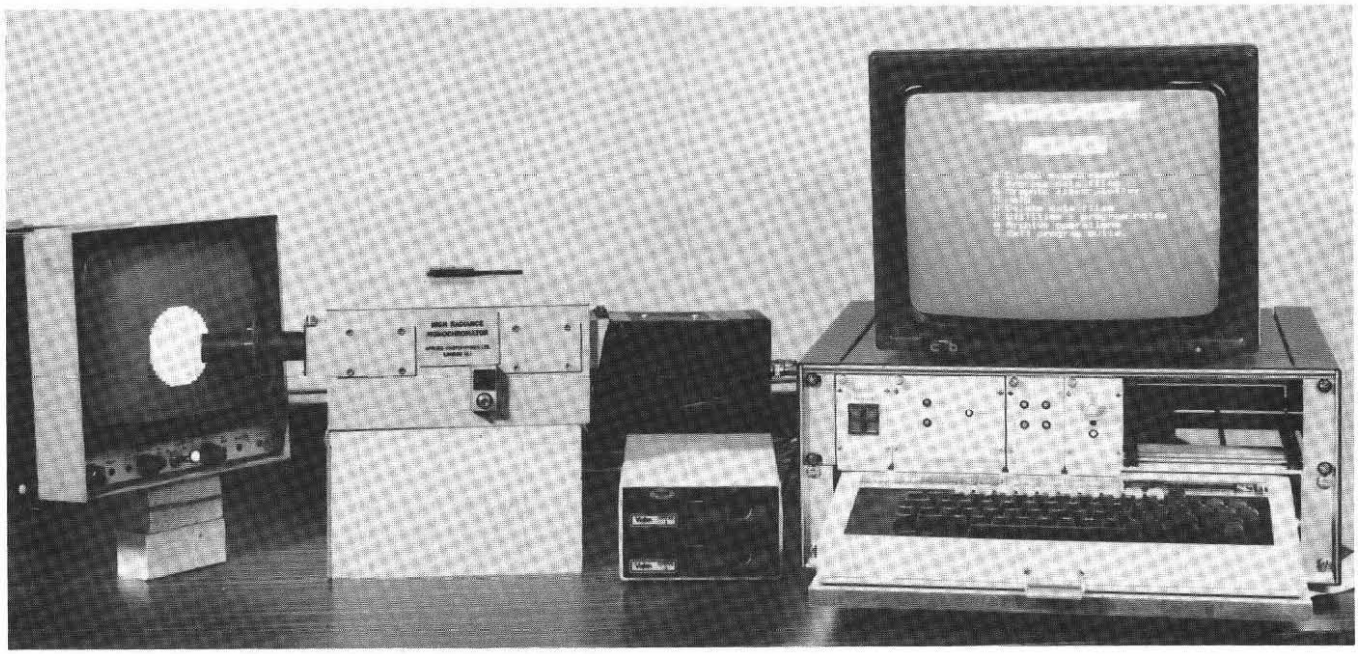

*Fig. 7 – The SpeCol equipment.*

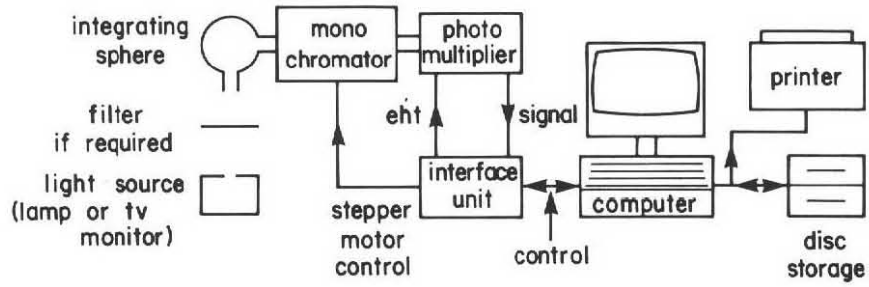


*Fig 8. – Block diagram of the SpeCol equipment.*

calibrated using a standard lamp, and the monochromator wavelength accuracy can be checked using discharge lamps, lasers or any other illuminant having well defined peaks or troughs in spectral power distribution. The system was designed to be a colorimeter and not a ***photometer***, in that it cannot measure luminance in absolute units (cd/m$^2$); however, if the e.h.t. supply to the photomultiplier is not changed between measurements it can be used to measure the relative luminance of sources, and hence it can measure filter performance.

Upon completion of a measurement scan the results are normally processed to remove the effect of the spectral gain characteristic of the monochromator and photomultiplier. The chromaticity coordinates in several colour spaces can then be derived. If the measurement is of a filter then the effects of the illuminant can be removed and replaced by those of another, perhaps notional, illuminant. The results can be plotted graphically and printed out as a spectral energy distribution or in the form of a chromaticity diagram in the CIE 1976 colour space. If the measurements being made are of a colour monitor then a triphosphor analysis can be performed, in which the colour rendering properties of the phosphors are calculated using a set of test colours. The results are tabulated using the CIELUV space and can be displayed graphically as well; details of the workings of this analysis are given in Appendix 2.

#### 4.1. Triphosphor Colorimetric Analysis

One of the most important uses of a colorimeter in television is for the assessment of the colour reproduction performance of colour picture monitors. A mathematical solution has been adopted

which involves measurement of the spectral radiance of the three display primaries and subsequent calculation of the displayed fidelity of a group of test colours. This process requires matrix arithmetic and a considerable amount of data minipulation. All these calculations are performed by the microcomputer that controls the measurement apparatus, thus resulting in 'stand alone' equipment.

The results of such a triphosphor analysis are shown in Figs. 9 and 10. Fig. 9 shows the spectral energy distribution of the three phosphors of a typical picture monitor, plotted with equal area under each curve to indicate the actual level required to produce an ***equal energy white***. Fig. 10(a) shows the chromaticity diagram for the same phosphors, with a triangle bounding the set of colours reproducible by standard phosphors. Also shown are parts of the spectrum locus and a set of test colours, displayed as a circle at each colour with a vector pointing to the colour position as displayed by those phosphors. More detail of these plots can be revealed by replotting on larger scales as is shown in Fig. 10(b).

It is important to take care in setting up the equipment before making a measurement, particularly in setting the gain of the detector by adjusting the e.h.t. It is advisable to check that the equipment can handle the range of incoming light levels, since the highest possible gain is required in order to fill the measurement range of the analogue-to-digital converter for greatest accuracy but it is easy to miss a high amplitude spike in a spiky spectrum.

## 5. SPECOL ACCURACY

### 5.1. Monochromator Bandwidth

The ***resolving power*** and therefore the bandwidth of the monochromator is limited by the spacing of the grating lines and the widths of the entry and exit slits. For this particular monochromator the minimum step size was 1 nm with a setting accuracy of 0.5 nm and the minimum bandwidth was measured as about 2.5 nm. This is acceptable for the measurement of phosphors and filters, but is inadequate for lasers and discharge lamps.

### 5.2. Photomultiplier Gain vs EHT

The measured photocurrent from the photomultiplier has a linear relationship with light but the gain of the device is logarithmically related to the applied e.h.t. For this particular photomultiplier tube the law was approximately:

$$Ip = k \,.\, \text{light} \,.\, V^{8.77}$$

where $Ip$ is the output current, $V$ is the applied e.h.t. and $k$ is a constant. Over the range of voltages used in this equipment (170 to 1100 volts) the gain changes by over $1.5 \times 10^6$:1 so it is essential that the e.h.t. does not drift during a measurement scan. Accordingly it is continually monitored by the computer. Clearly the system will be prone to noise when operated at highest sensitivity (highest e.h.t.) so filters have been incorporated in both the interface unit and the software; the group delay of these filters largely dictates the speed of the system.

### 5.3. Wavelength Accuracy

It is worth noting that the accuracy depends on wavelength. Fig. 11 shows the spectrum locus on a CIE 1960 diagram in which a vector of length 10 ***just noticeable differences*** (jnd) is represented as a length of 0.04 units of $u$ and $v$. Some 10 nm errors are shown on the spectrum locus and on the locus of television saturated colours, located on the triangle joining the coordinates of the phosphors. It is worth noting that the accuracy required to measure chromatic colours on the spectrum locus is considerably higher than that for even saturated colours in a television system. Small errors in wavelength setting would produce chromaticity shifts as tabulated below:

| | | chromatic colour | | saturated colour | |
|---|---|---|---|---|---|
| *colour* | *wave-length* | 1 nm error *in jnd* | 1 jnd error *in nm* | 1 nm error *in jnd* | 1 jnd error *in nm* |
| blue | 450 | 0.7 | 1.43 | 0.45 | 2.22 |
| cyan | 480 | 2.75 | 0.36 | 1.6 | 0.63 |
| green | 530 | 0.65 | 1.54 | 0.2 | 5.0 |
| yellow | 585 | 1.6 | 0.63 | 1.4 | 0.71 |
| red | 630 | 0.9 | 1.11 | 0.25 | 4.0 |

Thus a calibration accuracy of better than 0.6 nm is required of the measurement system for television purposes. However, a greater accuracy is always desirable for chromatic measurements.

One method of calibration uses optical filters, particularly Holmium and Didymium, which exhibit rapid changes in spectral density at precisely known wavelengths. The method of calibration is to measure the spectral transmittance distribution of the filter and then establish the measured wavelengths of the density maxima (transmittance minima) from the transmittance curves plotted at

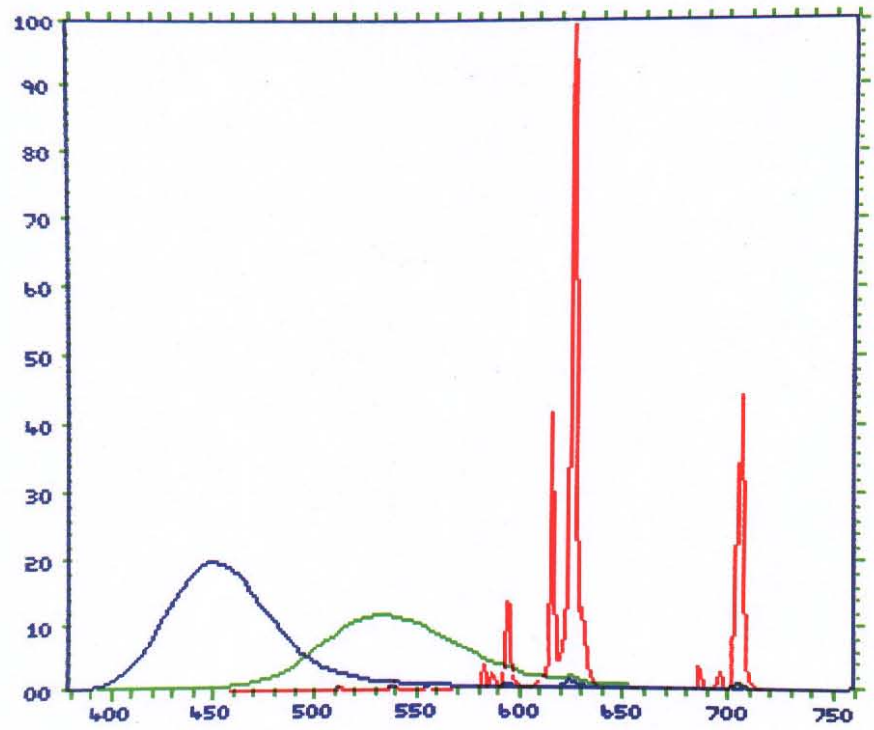


*Fig. 9 – Spectral output diagram of the phosphors of a typical colour monitor.*

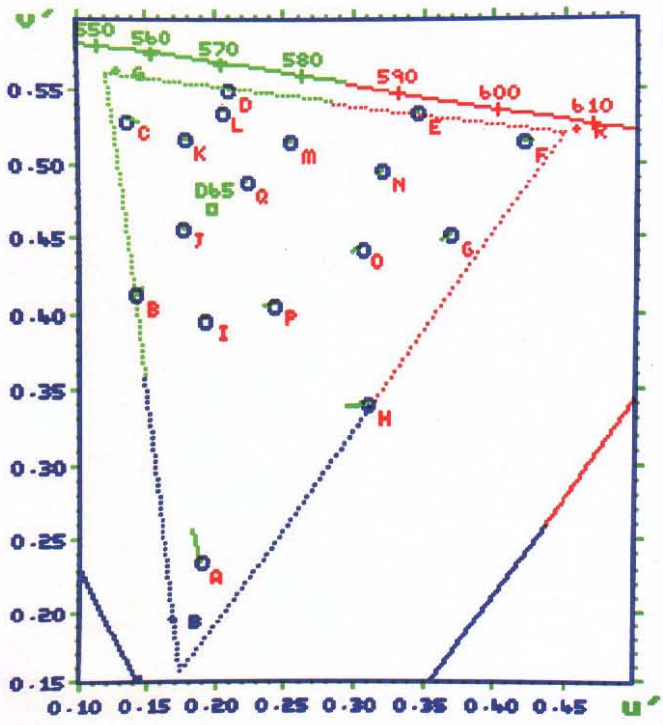


*Fig. 10(a) – Chromaticity diagram of a typical colour monitor showing test colour analysis. The phosphors are labelled* R, G *and* B*; test colours are indicated by a circle at the original colour and a vector pointing to the position of the displayed colour.*

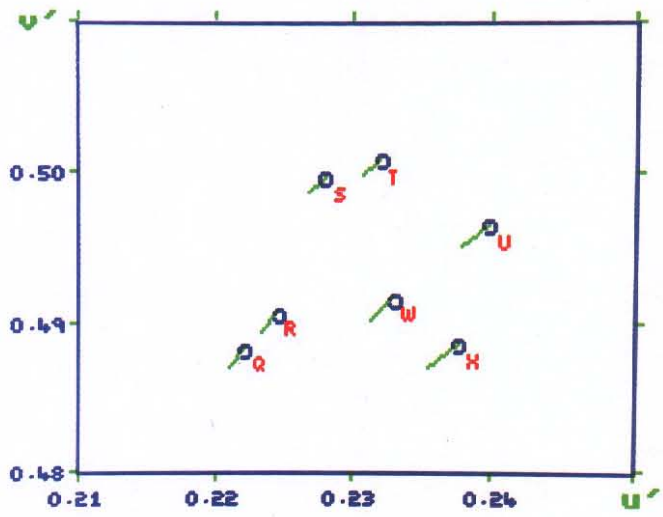


*Fig. 10(b) – An enlargement of a sensitive part of Fig. 10(a).*

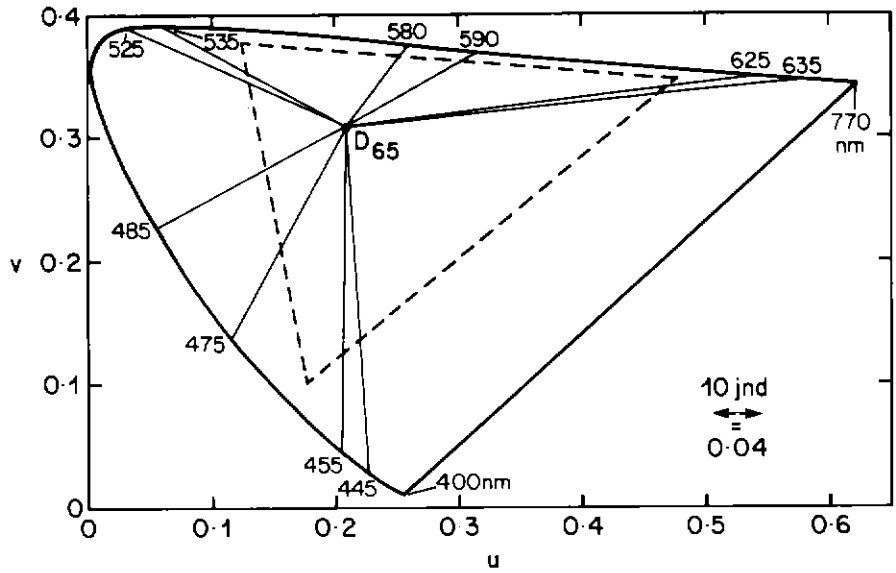


*Fig. 11 – The CIE 1976 chromaticity diagram, showing the magnitude of 10 nm errors at various wavelengths.*

1 nm intervals. These wavelengths are best calculated mathematically, since it is difficult to establish by eye directly from the curves where the minima are.

An alternative method uses light sources which have a spectral output containing well defined single lines, such as lasers, sodium lamps, mercury lamps or even fluorescent tubes. The result of measuring a laser emitting at 632.8 nm is shown in Fig. 12. From this it is impossible to determine by eye the wavelength of the peak with adequate accuracy since the bandwidth of the monochromator is too great. However, by calculating the chromaticity coordinates of the spectral colour and plotting it along with the spectrum locus[2] it is possible to calculate the wavelength of the laser by interpolation as shown in Fig. 13. The wavelengths of the red laser and of a sodium lamp were thus measured:

| Lamp | max energy | measured peak | precision | error |
|---|---|---|---|---|
| Laser | 632.8 | 632.31 | 0.06 | −0.49 |
| Sodium | 589.3 | 588.97 | 0.05 | −0.43 |

The precision quoted above is governed, to a large extent, by the quoted values for the spectrum locus, these being only available as published data to four significant figures in $x - y$ coordinates. These and other results show that the wavelength accuracy of the monochromator assembly is in error by −0.46 nm with a precision of 0.05 nm. Since the setting accuracy of the equipment is only to 0.5 nm

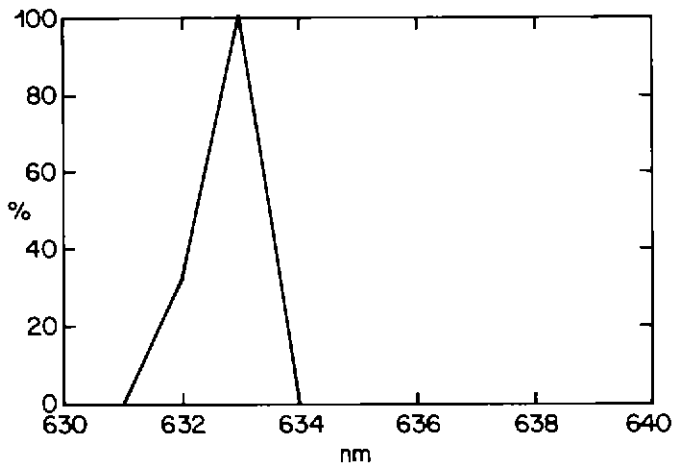


*Fig. 12 – The spectral emission diagram of a red laser.*

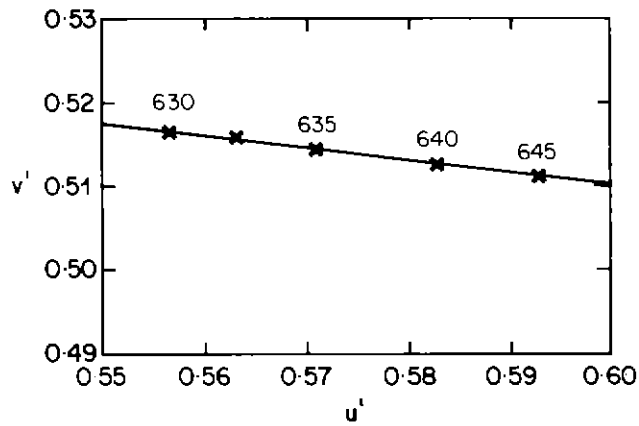


*Fig. 13 – Chromaticity diagram of a red laser.*

(the setting accuracy of the monochromator itself), this is perfectly acceptable, and can be allowed for in the calculations.

### 5.4. Colorimetric Precision and Short Term Accuracy

This was assessed by repeatedly measuring a large-filament tungsten lamp, previously calibrated by the National Physical Laboratory. At a certified current it radiated with a colour temperature of 2856 K to a tolerance of 5 K. Lamp current was monitored using a digital voltmeter to measure the drop across a stable resistor, and a period of over 60 minutes was allowed for the current to stabilise. Twenty nine measurements were made, over a period of four days, each being a scan from 380 to 760 nm.

Scans were done in groups at approximately 5 minute intervals each taking about 3 minutes to complete. The lamp current was observed to drift slightly (by less than 1 mA in each run) indicating that even one hour is insufficient time for the lamp and its housing to settle. Chromaticity coordinates were calculated and are shown plotted on a CIE 1931 diagram in Fig. 14 together with the ***Planckian locus*** which indicates the chromaticity coordinates of black body radiators. The coordinates of the mean of these measurements are plotted together with the calculated trend line. Clearly, the regression line and the Planckian locus are almost coincidental. Also shown is the calculated result that would be obtained if the gain of the measurement system were to drift by plus or minus 1.5% during a scan of the mean measurement; the two values fall almost on both lines. This indicates a strong likelihood that there may have been such a gain change occurring during some of the measurements, due either to lamp brightness variations or changes in the gain of the measurement system. The most probable cause of this is the e.h.t. supply to the photomultiplier system since a change of voltage of only 0.16% would be required to produce a 1.5% change of gain.

The distribution normal to the regression line probably indicates the precision or repeatability of the measurement process, giving a 95% probability that the indicated chromaticity is within 0.0001 of

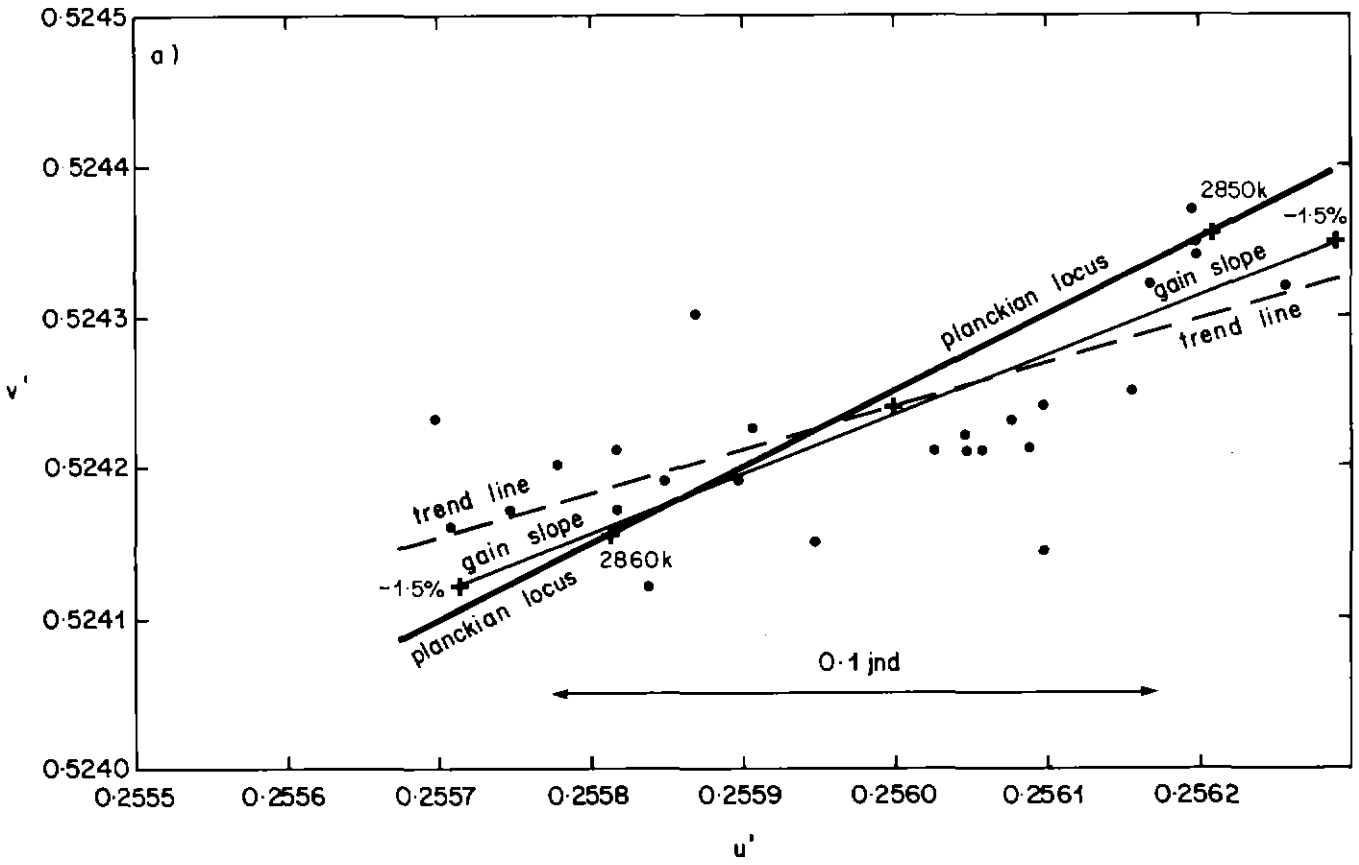


*Fig. 14 – Chromaticity diagram showing 29 measurements of a standard lamp, including the Planckian locus and regression line.*

the true value. The distribution along the regression line is 3.5 times greater; this indicates that there may be, in addition, brightness and gain variations as well as possible inaccuracies in setting of the lamp temperature. Taking the square root of the difference in variances, normal and parallel, to be the standard deviation of errors due to gain and lamp temperature changes alone gives a normalised figure of 0.000168; this implies a 95% probability that the lamp chromaticity is within 0.000336 of that of a Planckian body at 2856 K. This value represents 7.5 K along the Planckian locus, a value which is comfortingly close to the NPL specification of 5 K accuracy. It is not unreasonable to identify some of these errors as ambient temperature variations. It seems likely, therefore, that a considerable accuracy increase could be achieved, simply by controlling the temperature of the lamp housing to a greater degree, together with a more delicate control of the lamp current, and that closed loop control of e.h.t. would also afford some improvement.

These findings indicate that there is a 95% probability that measurements made in the CIE 1976 chromaticity system are within about 0.0002 of the true value, provided that gain drift and errors in the setting of the calibrating lamp can be eliminated. This value is approximately 0.05 jnd and is perfectly satisfactory for television purposes.

A subsidiary experiment was conducted to establish the relationship between lamp colour temperature and operating current. An old lamp was measured at various currents; the colour temperature of the lamp was obtained by interpolation and plotted vs lamp current in Fig. 15. The temperature coefficient is thus 467.5 K/A which reveals that the drift of lamp current noted above implies temperature changes of less than 1 K.

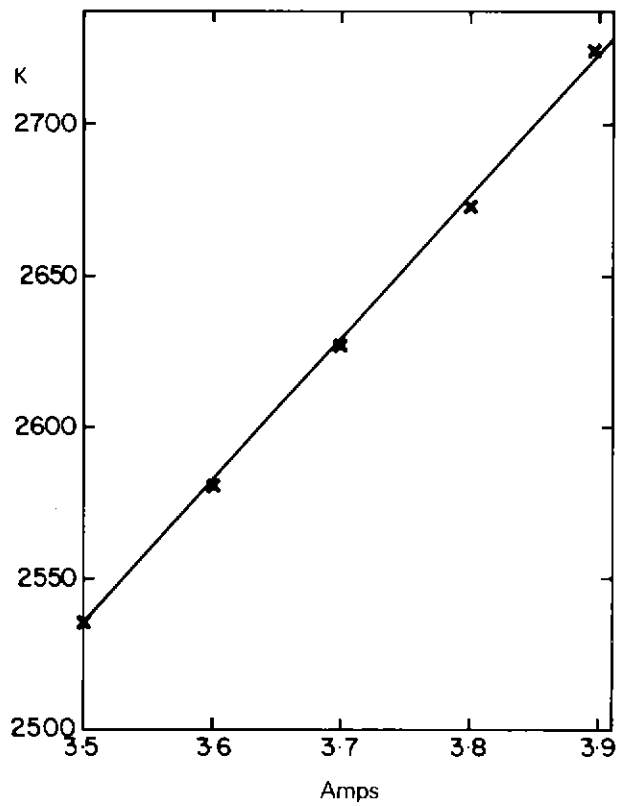


*Fig. 15 – The relationship between correlated colour temperature and filament current for a large filament tungsten lamp.*

## 6. SPECOL LIMITATIONS AND DIFFICULTIES

Practical experience has highlighted several minor points which can affect the way in which the system is best used.

### 6.1. Monitor Measurements

When measuring the performance of a monitor it is desirable to lock the field timebase to the mains power-supply frequency. This ensures that any mains related brightness modulation does not move on the picture during a measurement scan. Typical problems which might occur are scan geometry changes and video signal level modulation, although neither of these effects should be apparent on a high quality studio monitor. Ideally only a small part of the raster should be illuminated to avoid shading and the light should be integrated spatially by collection into an integrating sphere or onto a white tile.

Practical experience has shown that the colour of a phosphor is dependent upon several factors; changes in e.h.t., brightness, and temperature all produce colour shifts. A series of measurements of four similar monochrome monitors exhibiting effects due to one or more of these causes is shown in Fig. 16. All four were measured eight times on one day, and one was repeated eight times on the following day. Each monitor was allowed more than one hour to settle before measurement. Each measurement took approximately three minutes. Clearly the measurements do not have a random scatter but fall on a series of regression lines. The cause of this is probably due to measurement system gain and/or brightness changes. A measurement representing the centroid of each group has been recalculated to simulate gain drift of 1·5% during measurement as with the lamp measurement of Section 5.4; clearly these points lie on or near the regression lines which suggests that gain changes may have occurred during the real measurements. It

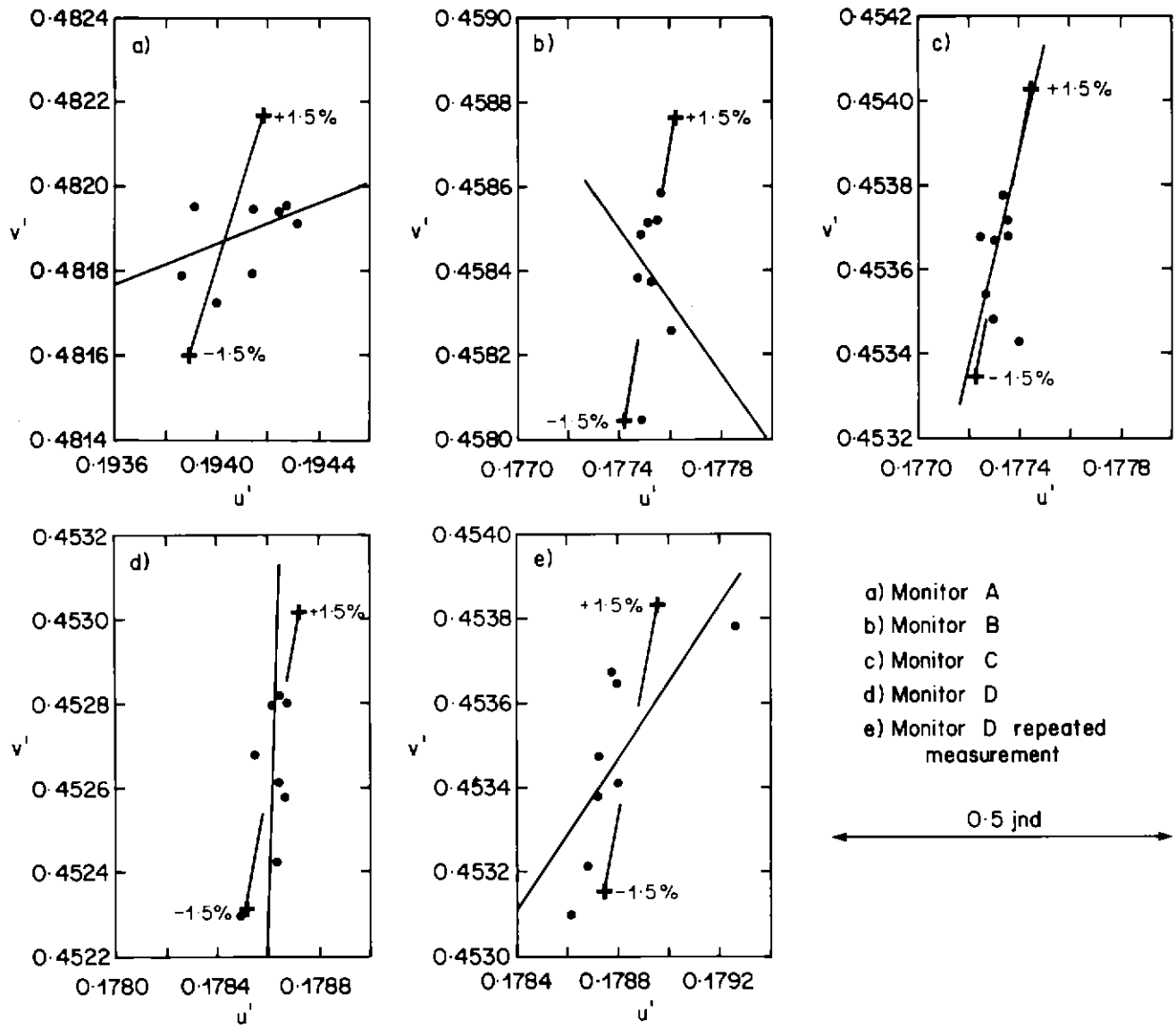


*Fig. 16 – Enlarged chromaticity diagrams for 4 monochrome monitors, showing regression lines and the calculated effects of gain drift during measurement.*

is considered unlikely that a monochrome monitor would be stable to better than 1% under normal conditions and that therefore this problem may be incurable. To some extent these observations confirm the conclusions drawn in Section 5 about the precision of the equipment.

It is evident that monitor measurements should not be made singly. Instead it is suggested that several should be completed in succession, each separately processed to calculate the chromaticity coordinates and an average made of the central members of the group. Such an average should be made of the original spectral data, resulting in a mean spectrum which can then be processed to find the true chromaticity.

When measuring colour monitor phosphors another problem arises. Since the red phosphor has a spiky spectrum, as can be seen in Fig. 9, in order to catch the fine detail of the spectrum the scan must be done at 1 nm intervals; consequently a complete scan can take about 15 minutes and making eight measurements would take up to two hours during which time drift can take place in both the monitor and the measuring equipment. There is no solution to this, and some care is required. It is, however, unwise to take averages of spiky spectra, since the fine detail may be lost; it is better to use the single measurement which occurs at the centre of the chromaticity spread, or even to rely on a single measurement.

### 6.2. Filter Measurements

When measuring the transmittance or reflectance of a filter it is necessary to know the spectral

power distribution of the illuminating source. Any broad spectrum lamp will do, but it must be stable. For a short series of measurements it is adequate to measure the lamp before and after the tests and produce a mean spectrum of the lamp for use in the processing. For a longer series either the lamp must have assured stability or rechecking must be more frequent. In practice only very small changes in lamp spectrum were noted if the lamp was allowed more than one hour to settle. It is possible to calculate density (D) for neutral filters using the output of the SpeCol equipment by making a calculation of the area under the spectral curve:

$$D = \log_{10}\text{area}_i - \log_{10}\text{area}_f$$

where the subscripts $i$ and $f$ refer to the illuminant and the filter respectively. With care it is possible to measure densities up to about 2.5.

#### 6.3. Discharge Lamp Measurements

Although the SpeCol equipment was designed primarily for the measurement of television displays, it can also be of use in measuring the spectral power distribution of lamps, including some arc discharge lamps. These typically have a spiky spectral power distribution extending over more than one octave, from near ultraviolet into near infrared, and this highlights a problem which occurs in monochromators – the generation of subharmonic spectra.

The dispersive grating in the monochromator spreads the incoming light into the wanted primary rainbow together with a series of unwanted secondary ones emerging at greater angles. The effect of the second order rainbow is that light radiated at, for example, 250 nm manifests itself at the same position as the first order diffracted light at 500 nm. Higher orders of diffraction are usually not significant. One solution is to insert an ultraviolet absorbing filter in the light path once the monochromator has stepped beyond about 450 nm. Unfortunately this results in some light loss since about 2% is lost by reflection at each glass surface. In order to preserve calibration it is desirable to include a nonabsorptive filter in the light path when the UV filter is not present; alternatively the response of the UV filter at each wavelength can be calibrated and used to compensate the measurements taken via this filter.

## 7. CONCLUSIONS AND USEFUL BYPRODUCTS OF THE SPECOL STUDY

#### 7.1. SpeCol Practical Uses

At the time of writing this Report, the equipment has successfully made over 200 measurements on a range of lamps, filters and monitors. Lamp measurements have been made on tungsten and arc discharge types, to assess their suitability for use as television lighting. Filter measurements have been made on transmissive, neutral density filters, and reflective materials for use in test charts.

Monitor measurements have been made to test the accuracy and consistency of monochrome monitors, particularly with respect to matching monochrome to colour units with a D65 balance point, and to assess the evenness of screen illumination. Colour monitor measurements have been made to assess the colour rendering accuracy of new monitors.

#### 7.2. Byproducts of the Work

There have been a number of significant byproducts of this work. Some arose from the recalculation of accepted colorimetric formulae, others came from measurements made with the finished equipment.

The specification of chromaticity coordinates for EBU phosphors was originally made in the CIE 1931 colour space and specified to two decimal places. It has been common practice to use a three figure accuracy for conversion of these data expressed in the CIE 1960 colour space. Both sets are tabulated below:

| Colour | $x$ | $y$ | $z$ | $u$ | $v$ | $w$ |
|---|---|---|---|---|---|---|
| red | 0.64 | 0.33 | 0.03 | 0.451 | 0.349 | 0.200 |
| green | 0.29 | 0.60 | 0.11 | 0.121 | 0.374 | 0.505 |
| blue | 0.15 | 0.06 | 0.79 | 0.175 | 0.105 | 0.720 |
| white | 0.3127 | 0.3290 | 0.3583 | | | |

where white is taken to be illuminant D65, the phase of daylight which correlates with a Planckian body radiating at 6504 K. The $xyz$ values are a specification of the chromaticities of the primaries, but the $uvw$ values were derived from them and then rounded to those significant figures. Thus they are not as accurate as the $xyz$ values. Throughout the work on SpeCol the original $xyz$ data have been used for all calculations.

All SpeCol colour differences calculated in colour performance assessments are done in the CIE 1931 space, the results being displayed in the CIELUV space together with conventional jnd values. The EBU has recommended the use of the CIELUV error space for television colorimetry, and experience so far has shown that it agrees well with jnd measurements; the comparison has revealed the relationship that one jnd unit approximately equals five $\Delta E^*$ psychometric units.

The chromaticity specification for the BBC 'Standard white' monochrome monitor has hitherto been:

$$x = 0.313 \qquad y = 0.344$$

this being the measurement of a monochrome tube typical of the 'D65' tubes available to the BBC. Such monitors are used in studio areas alongside colour monitors where it is important that the balance point of all monitors is the same. Recent re-measurements of the 'standard' tube have revealed the slightly different values:

$$x = 0.3203 \qquad y = 0.3534$$

The differences are worryingly greater than expected, but the recent measurements are repeatable and there appears to be no readily-identifiable error. They have been tentatively adopted as an updated specification for the standard, but further work will be done to confirm the new values.

Measurements have been made of the brightness uniformity of monochrome tubes to establish the acceptability limits. The results are intended to form a BBC specification but are as yet unpublished. Further work is planned to establish the brightness and colour shift limits for colour monitors as part of an EBU program aimed at creating a recommendation for guidance of manufacturers.

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## APPENDIX 1

### Basics of Colour Analysis

The principle of additive colour mixing to produce one colour by adding three others is an expression of ***Grassman's First Law***, which states that any four colours are linearly dependent, so any colour Q can be expressed in the form:

$$Q = rR + gG + bB$$

where $R$, $G$ and $B$ are any three different colours and $r$, $g$ and $b$ are scalar quantities, provided that it must not be possible to derive any of the primary colours from a combination of the other two. ***Grassman's Second Law*** can be used to describe further the principle of colour mixing in which any two colours $Q_1$ and $Q_2$, specified by a mixture as described above, when themselves mixed, can be expressed in the form:

$$Q = Q_1 + Q_2 = (r_1 + r_2)R + (g_1 + g_2)G + (b_1 + b_2)B$$

As a corollary to these statements, since colours can be manipulated linearly it is possible to transform a

colour equation in one set of primaries into an equation in another set by linear matrix arithmetic. Thus the choice of primaries for any colour system is one of convenience.

The original CIE (***Commission Internationale d'Eclairage***) set of primaries denoted $R$, $G$ and $B$ were monochromatic sources radiating at 700, 546.1 and 435.8 nm respectively; the reasons for selection of these wavelengths need not be discussed here. Using the expression of Grassman's First Law given above, it is possible to describe any colour by a mixture of these primaries, given that equal amounts produce white.

The amounts of each primary required to match a colour are called the ***tristimulus values***; they completely describe the appearance of the colour, but in a way which is of little use for most purposes since negative amounts of red are required in order to describe some green colours. This seemingly impossible situation is resolved in an optical colorimeter by diverting the red primary onto the test colour sample in order to produce a match, and taking the amount of red thus used as a negative quantity. The locus of all spectral colours is shown in Fig. 3, plotted in a plane of constant luminance through a colour space using, as axes, the ***chromaticity coordinates*** derived by normalising the tristimulus values:

$$r = \frac{R}{R + G + B}$$

$$g = \frac{G}{R + G + B}$$

$$b = \frac{B}{R + G + B}$$

so that

$$r + g + b = 1$$

The chromaticity coordinates completely describe the colour appearance, but not the luminance, of the colour. It can be seen that for a large part of the spectrum the red coordinate is negative, not a very helpful situation since this implies that negative red light is required to represent those colours.

A better colour space can be derived from the $R\ G\ B$ data by transforming into the $X\ Y\ Z$ colour space. This assumes representation of colours by a mixture of three new primaries, $X$ $Y$ and $Z$, which lie outside the spectrum locus and are thus unreal. The $Y$ primary is chosen to represent the luminance of the colour, and thus the two other primaries lie on the plane of zero luminance and describe the colour. The mathematical transformation is a simple matrix equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0{\cdot}48999 & 0{\cdot}31001 & 0{\cdot}20000 \\ 0{\cdot}17696 & 0{\cdot}81240 & 0{\cdot}01604 \\ 0{\cdot}00000 & 0{\cdot}01000 & 0{\cdot}99000 \end{bmatrix} \cdot \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

where the coefficients have been chosen such that $X = Y = Z$ for an equi-energy illuminant. Since these primaries cannot exist as real illuminants the simple optical colorimeter cannot be used and another method is required to produce tristimulus values in this colour space. This is most easily done by measuring the spectral power distribution of the colour ($P_\lambda$), multiplying it by the ***colour matching functions*** for each of the primaries ($\bar{x}$, $\bar{y}$ and $\bar{z}$) wavelength by wavelength, and integrating the product curve to obtain the values $X$ $Y$ and $Z$. These colour matching functions are shown in Fig. 5. Thus:

$$X = \int P_\lambda \bar{x}_\lambda \, d\lambda \quad Y = \int P_\lambda \bar{y}_\lambda \, d\lambda \quad Z = \int P_\lambda \bar{z}_\lambda \, d\lambda$$

where the integration is over the entire range of visible light wavelengths.

The spectrum locus is shown again in Fig. 2, plotted in a plane of constant luminance using, as axes, the

$x$ and $y$ chromaticity coordinates derived, again, by normalising the tristimulus values:

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

so that

$$x + y + z = 1$$

A significant problem with the CIE $xyz$ colour space is that the length of a vector in the $xy$ plane which joins together two colours which are only just distinguishable from each other is a function of its location in that plane. This has been established by many workers in colorimetry, and there have been many quests for a new colour space, transformed from CIE $xyz$ values, in which these vectors are of equal length. Fig. 17 shows this effect in the CIE 1931 chromaticity diagram illustrating the ***MacAdam ellipses***,[4] whose radii represent the ease of identifiying colour differences. Note that the range of sizes of the ellipses is about forty to one.

The aim of the inventors of all chromaticity diagrams, derived from the CIE 1931 system by linear transformation, is to render these ellipses as equal sized circles. The ***CIE 1960 Uniform Colour Space***, suggested by MacAdam (1937) was accepted by the CIE in 1960. The transformation is into new tristimulus values, $U$, $V$ and $W$, and subsequently into chromaticity coordinates $u$ and $v$ which may be derived directly from $x$, $y$ and $z$, or from just $x$ and $y$:

$$u = \frac{2x}{6y - x + 1.5} \qquad v = \frac{3y}{6y - x + 1.5}$$

($w$ can be derived from these values since $u$, $v$ and $w$ sum to unity). Fig. 18 shows the MacAdam ellipses in this diagram, which exhibit a much smaller range of sizes.

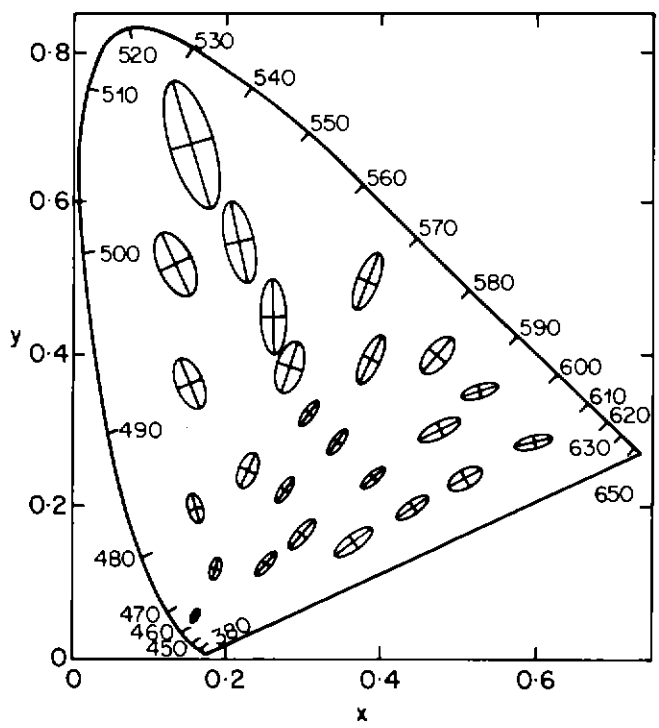


*Fig. 17 – The visibility of colour errors in the CIE 1931 diagram, each ellipse shows the magnitude of equally visible colour errors (MacAdam ellipses).*

More recently, after suggestion by Eastwood (1974) the $v$ coordinate was increased by 50% to form $v'$ in order further to improve the situation to form the ***CIE 1976*** chromaticity diagram; the values $u'$ and $v'$ can thus be derived:

$$u' = \frac{U'}{U' + V' + W'} = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$

$$v' = \frac{V'}{U' + V' + W'} = \frac{9Y}{X + 15Y + 3Z} = \frac{9y}{-2x + 12y + 3}$$

A number of nonlinear transforms have been suggested, for instance by Farnsworth[5] and MacAdam,[6] in which these ellipses are truly equal sized circles, but have not been adopted.

This search for a perfect system, in which a ***just noticeable difference*** (jnd) between two colours is of uniform size all through the colour space, is probably doomed to be unsuccessful. However, it has led to a subjective definition of the jnd:

for chromaticity differences: a vector of length 0.004 in $u\ v$ units
for luminance differences: a 2% difference

Under perfect viewing conditions an experienced observer can perceive differences of about one tenth of these values.[8]

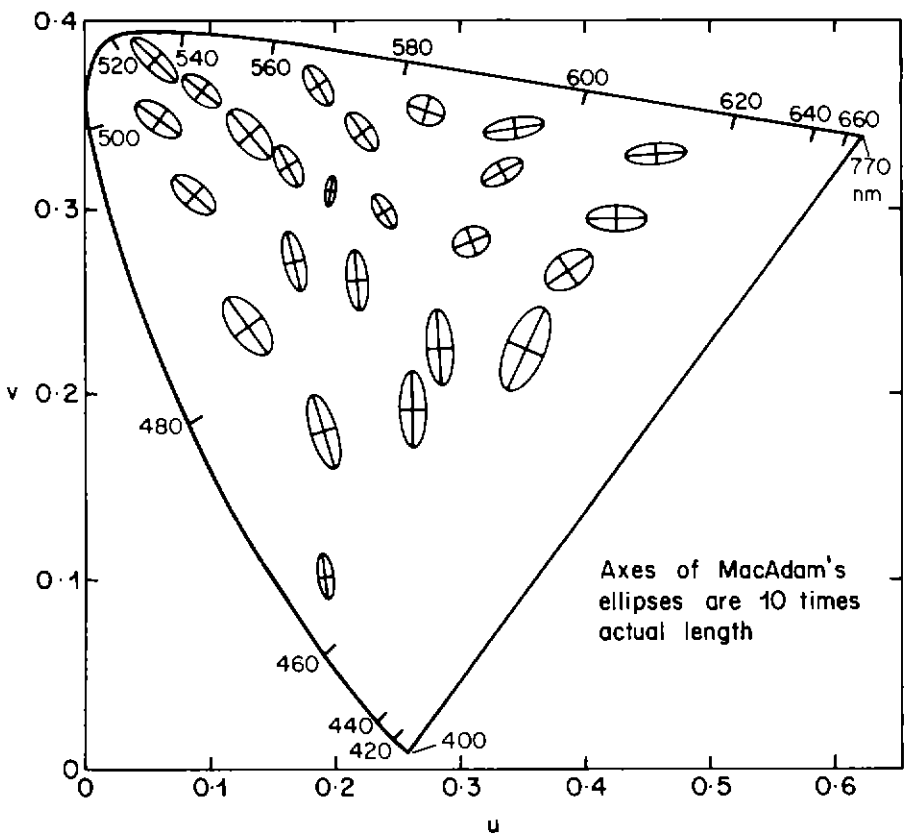


*Fig. 18 – The MacAdam ellipses redrawn on a CIE 1960 diagram, showing greater uniformity of ellipse size.*

The chromaticity difference is defined as:

$$\Delta uv = [(u_a - u_b)^2 + (v_a - v_b)^2]^{\frac{1}{2}}$$

and the luminance difference is defined as:

$$\Delta Y = \log_e Y_a - \log_e Y_b$$

where subscripts $a$ and $b$ refer to the two colours in question. The total colour difference in jnds is then measured as the r.m.s. sum of the colour and luminance values, each normalised by their jnd calibrations:

$$n = [(\Delta uv/0.004)^2 + (\Delta Y/0.02)^2]^{\frac{1}{2}}$$

this system was the basis of colorimetric measurement at Research Department for several years.

The CIE has formulated another space, called the ***CIELUV 1976*** system, which is well suited to the needs of television measurements. This attempts to express mathematically the relationship between a colour and its illuminant, mimicking the perception process as far as possible. The values in this space have been formulated to correspond closely with the subjective appearance of the colour; thus $L^*$ is the ***psychometric lightness, chroma***, ($C^*$) is related to saturation and is the magnitude of the vector formed by $u^*$ and $v^*$, and hue ($H^*$) is the angle formed by $u^*$ and $v^*$. The definition of these is

$$L^* = 116\,(Y_d/Y_o)^{\frac{1}{3}} - 16$$
$$u^* = 13\,L^*(u'_d - u'_o)$$
$$v^* = 13\,L^*(v'_d - v'_o)$$

where the subscripts $d$ and $o$ refer to the displayed colour and the illuminant. The formulae are empirical but appear to be well suited to television measurements.

The difference between the values for two colours can then be expressed:

$$\Delta L^* = L^*_a - L^*_b$$
$$\Delta u^* = u^*_a - u^*_b$$
$$\Delta v^* = v^*_a - v^*_b$$

where subscripts $a$ and $b$ refer to the two colours being compared. The total colour difference equation is then simply:

$$\Delta E^* = (\Delta L^{*2} + \Delta u^{*2} + \Delta v^{*2})^{\frac{1}{2}}$$

Practical experience with this system has shown that five units of $\Delta E^*$ are approximately equal to one jnd although this relationship is not linear.

One further difficulty arises from the fact that, for television purposes in PAL system I, the three primaries $R$ $G$ and $B$ are not those of the CIE definition, but are those of typical phosphors used in television cathode ray tubes. Measurements must relate, therefore, to them instead. They are defined in the $xy$ chromaticity plane as follows:

| colour | $x$ | $y$ | $z$ |
|---|---|---|---|
| red | 0.64 | 0.33 | 0.03 |
| green | 0.29 | 0.60 | 0.11 |
| blue | 0.15 | 0.06 | 0.79 |
| white | 0.3127 | 0.3290 | 0.3583 |

where the white balance point is illuminant D65. This is the mean daylight illuminant; its chromaticity does not lie on the locus of Planckian black body radiators and thus it does not have a true colour temperature but has a correlated colour temperature of 6504 K. In order to obtain the matrix equation which relates

$XYZ$ tristimulus values to television (EBU) $RGB$ tristimuli we must mathematically balance the phosphors to white and solve the matrix equation:

$$\begin{bmatrix} 0.64 & 0.29 & 0.15 \\ 0.33 & 0.60 & 0.06 \\ 0.03 & 0.11 & 0.79 \end{bmatrix} \cdot \begin{bmatrix} a_r \\ a_g \\ a_b \end{bmatrix} = \begin{bmatrix} 0.9505 \\ 1.0000 \\ 1.0891 \end{bmatrix}$$

where the '$a$' matrix is the phosphor multiplier required to balance the phosphors to the normalised white in the right hand side of the equation. This operation is the equivalent of balancing a colour monitor by adjusting the gains to match to D65; the 'a' matrix holds the resulting gain values for $R$ $G$ and $B$.

This yields a new tristimulus matrix

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4306 & 0.3415 & 0.1784 \\ 0.2220 & 0.7067 & 0.0713 \\ 0.0202 & 0.1295 & 0.9394 \end{bmatrix} \cdot \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

and its inverse can be derived to obtain $R$ $G$ and $B$ from $X$ $Y$ and $Z$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.0627 & -1.3928 & -0.4759 \\ -0.9689 & 1.8756 & 0.0417 \\ 0.0677 & -0.2286 & 1.0690 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

These are the normally accepted matrices, however, in order to obtain the highest possible accuracy, these matrix equations have been recalculated without rounding of the data values, and are given below:

$$\begin{bmatrix} 0.430554133 & 0.341549949 & 0.178352386 \\ 0.222004475 & 0.706655066 & 0.071340954 \\ 0.020182225 & 0.129553429 & 0.939322564 \end{bmatrix} \cdot \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

and

$$\begin{bmatrix} 3.063359317 & -1.393389643 & -0.475823886 \\ -0.969243726 & 1.875967206 & 0.041555540 \\ 0.067861518 & -0.228799672 & 1.069089666 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Differences may be noted between these and the standard values, given above, which are entirely due to rounding errors. The inverse matrix exhibits greater differences than the first matrix since rounding errors have not been compounded as in the derivations of the original matrices. These two matrices are consistent and have been used in all the data analysis.

## APPENDIX 2

### Test Colour Analysis

In order to analyse the colour performance of a tri-phosphor display it is necessary first to convert the tristimulus values of each test colour from EBU primary $R$ $G$ $B$ values to CIE 1931 $X$ $Y$ $Z$ primary values, modify them according to the new phosphors, and then convert to chromaticity coordinates in CIE 1976 $Y$ $u'$ $v'$ space. To do this, several stages are involved:

1. *Transform D65 chromaticity coordinates into xyz values*

The chromaticity coordinates (subscript $o$) of the balance point of the system, D65, are stored for reference in $u'$ $v'$ values; these must be converted to $xyz$ coordinates:

$$x_o = 9u'_o/(6u'_o - 16v'_o + 12)$$
$$y_o = 4v'_o/(6u'_o - 16v'_o + 12)$$
$$z_o = 1 - x_o - y_o$$

2. *Transform phosphor chromaticity coordinates into xyz space*

The chromaticity coordinates of the three phosphors (subscript $p$) are similarly transformed into $xyz$ chromaticities.

$$x_p = 9u'_p/(6u'_p - 16u'_p + 12)$$
$$y_p = 4v'_p/(6u'_p - 16v'_p + 12)$$
$$z_p = 1 - x_p - y_p$$

3. *Obtain the tristimulus values of the phosphors*

This is the direct equivalent of balancing a monitor to D65, and must be done before any analysis can be started. The procedure is rather complex. As was described in Appendix A, the tristimulus values for a colour $n$ are obtained, using its $R\ G\ B$ values, from

$$\begin{bmatrix} X_n \\ Y_n \\ Z_n \end{bmatrix} = \begin{bmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{bmatrix} \cdot \begin{bmatrix} R_n \\ G_n \\ B_n \end{bmatrix}$$

which uses the $X\ Y\ Z$ tristimulus values of the phosphors, but these are not yet known. They can be derived from the phosphor chromaticities by balancing them to D65, since

$$\begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix} \cdot \begin{bmatrix} a_r \\ a_g \\ a_b \end{bmatrix}$$

where the '$a$' matrix gives multiplying factors for the phosphors. This equation can be solved since the tristimulus matrix on the left hand side of the equation is that of D65; thus

$$\begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} = \begin{bmatrix} x_o/y_o \\ 1 \\ z_o/y_o \end{bmatrix}$$

and by definition

$$R_o = G_o = B_o = 1$$

Now the '$a$' matrix can be found by matrix inversion,

$$\begin{bmatrix} a_r \\ a_g \\ a_b \end{bmatrix} = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix}^{-1} \cdot \begin{bmatrix} x_o/y_o \\ 1 \\ z_o/y_o \end{bmatrix}$$

and the '$a$' values can then be substituted into the tristimulus equation to produce the transformation matrix by multiplying each chromaticity coordinate by the appropriate '$a$' factor. This tristimulus matrix is then used for colour analysis.

4. *Transform test colour chromaticities*

First $Y\ u'\ v'$ source coordinates (subscript $s$) of the test colour are converted to $R\ G\ B$ values using the relationships described in Appendix 1. The chromaticity coordinates are given by

$$x_s = 9u'_s/(6u'_s - 16v'_s + 12)$$
$$y_s = v'_s/(6u'_s - 16v'_s + 12)$$
$$z_s = 1 - x_s - y_s$$

from which the tristimulus values can be derived

$$X_s = x_s Y_s/y_s$$
$$Z_s = z_s Y_s/y_s$$

and the $R\ G\ B$ values found

$$\begin{bmatrix} R_s \\ G_s \\ B_s \end{bmatrix} = \begin{bmatrix} 3.063359317 & -1.393389643 & -0.475823886 \\ -0.969243726 & 1.875967206 & 0.041555540 \\ 0.067861518 & -0.228799672 & 1.069089666 \end{bmatrix} \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}$$

Then the display coordinates (subscript $d$) of the test colour can be calculated using the phosphor transformation matrix derived in Section 3 above, thus the tristimulus values are

$$\begin{bmatrix} X_d \\ Y_d \\ Z_d \end{bmatrix} = \begin{bmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{bmatrix} \cdot \begin{bmatrix} R_s \\ G_s \\ B_s \end{bmatrix}$$

from which the chromaticity coordinates can be calculated

$$u'_d = 4X_d/(X_d + 15Y_d + 3Z_d)$$
$$v'_d = 9Y_d/(X_d + 15Y_d + 3Z_d)$$

These are then the final display coordinates of the test colour. Using the relationships described in Appendix 1 the colour errors can be derived from the source and display chromaticity coordinates in both jnd and $\Delta E^*$ units for further use.



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